

Heavy Quark Analogues of the Θ and Their Excitations

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Predictions for the low-lying excitation spectrum of positive parity pentaquark systems containing one \bar{c} or \bar{b} antiquark and four light u, d quarks are obtained in the quark model picture for models with spin-dependent interactions given either by effective color magnetic (CM) exchange or effective Goldstone boson (GB) exchange. For the GB model, 4 excited states are predicted to lie within $\simeq m_\Delta - m_N$ of the $J^P = 1/2^+$ ground state while, for the CM model, 10 states are expected in the same range. Both the lowest excitation energy and the relative splittings are much smaller in the CM case. These predictions are on the same footing as those for the analogous splittings in the non-exotic baryon sector and, as such, provide a means of not only testing the models, but potentially ruling out either one, or both.

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I. INTRODUCTION

Interest in “heavy pentaquarks” (states with the quark content $\bar{Q}q^4$, where $\bar{Q} = \bar{c}$ or \bar{b} , $q = \ell$ or s , with $\ell = u$ or d) goes back more than fifteen years to observations made in the context of the effective color-magnetic exchange (CM) model, a model with considerable phenomenological success in describing splittings in the baryon spectrum. The spin-dependent interactions of the model have the form

$$H_{CM} = \sum_{i < j} \frac{C_{CM}}{m_i m_j} f_{CM}(\vec{r}_{ij}) \vec{\sigma}_i \cdot \vec{\sigma}_j \vec{F}_i \cdot \vec{F}_j, \quad (1)$$

where $\vec{\sigma}_i$ are the Pauli spin matrices, $\vec{F}_i = \vec{\lambda}_i/2$ for quarks and $\vec{F}_i = -\vec{\lambda}_i^*/2$ for antiquarks, are the Gell-Mann color matrices, C_{CM} is a constant, m_i is the constituent quark mass, and $f_{CM}(\vec{r}_{ij})$ contains the spatial dependence, usually taken to be a smeared version of the delta function. In the combined $SU(3)_F$ and $m_{\bar{Q}} \rightarrow \infty$ limits, the $J^P = 1/2^-$, flavor 3_F heavy pentaquark channel was found to have a hyperfine expectation optimally attractive relative to the corresponding strong decay threshold, BP_H (with B and P_H the relevant octet baryon and heavy pseudoscalar meson) [1]. Subsequent investigations

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showed that $SU(3)_F$ breaking, kinetic energy, confinement, and $m_{\bar{Q}} \neq \infty$ effects all reduced binding. The combined effect made it unlikely that even the most favorable of the $J^P = 1/2^-, 3_F$ states (that with isospin and strangeness $(I, S) = (1/2, -1)$) would bind [2]. Since an attractive s-wave interaction insufficiently strong to bind produces positive phase motion, but not resonant behavior, such a $J^P = 1/2^-$ state, if above threshold, would be non-resonant, with an s-wave “fall-apart” decay to ND_s or NB_s .

The situation is rather different for the Goldstone boson exchange (GB) model, where negative parity heavy pentaquark states were found to be unbound by several 100 MeV [5]. The model involves effective interactions generated by Goldstone boson exchange, and was introduced to deal with certain phenomenological problems of the CM model, in particular the problem of the incorrect ordering of positive and negative parity excited baryon states [3]. The effective spin-dependent GB qq interactions have the form

$$H_{GB} = \sum_{i < j} \frac{C_{GB}}{m_i m_j} f_{GB}(\vec{r}_{ij}) \vec{\sigma}_i \cdot \vec{\sigma}_j \vec{F}_i \cdot \vec{F}_j, \quad (2)$$

with $\vec{\sigma}_i$ the Pauli spin matrices, $\vec{F}_i = \vec{\lambda}_i$ the Gell-Mann $SU(3)_F$ flavor matrices, and C_{GB} a constant. The explicit form of $f_{GB}(\vec{r}_{ij})$, whose parameters are fixed from the study of the baryon spectrum, is given in Ref. [4]. As first noted by Stancu [4], reducing the q^4 orbital permutation symmetry from $[4]_L$ to $[31]_L$ by introducing a single p-wave excitation among the four light quarks increases the hyperfine attraction. The interaction turns out to bind those positive parity ($P = +$), 6_F states having quark content $\bar{Q}\ell^4$ and discrete quantum numbers $(I, S, J_q) = (0, 0, 1/2)$ (where J_q is the total intrinsic spin which, combined with the orbital $L = 1$, yields the total J). The binding energies are 76 MeV and 96 MeV for the $\bar{Q} = \bar{c}$ and \bar{b} systems, respectively [4]. The other members of the 6_F multiplet are predicted to be unbound, but by less than ~ 100 MeV, and hence might appear as genuine resonances, since an attractive interaction in a relative p-wave can play off against the peripheral centrifugal repulsion to produce resonant behavior.

An experimental search for strong interaction stable anticharmed $(I, S) = (1/2, -1)$ pentaquark states decaying to $K^{*0}K^-p$ was performed by the E791 Collaboration [6]. Negative results were reported in the mass range 2.75 – 2.91 GeV.

The discovery of the $S = +1$ θ baryon [7] has led to a revived interest in heavy pentaquarks [9, 10, 11, 12, 13, 14, 15, 16, 17]. If, as is now generally assumed, the θ parity is positive, then whatever mechanism makes the θ narrow is likely to also make its heavy quark analogues, the $\theta_{c,b}$, narrow. The H1 collaboration has recently presented evidence for a narrow anticharmed pentaquark resonance, with mass $3099 \pm 3 \pm 5$ MeV and width compatible with experimental resolution, decaying to $D^{*-}p$ [9]. This observation has yet to be confirmed. The H1 state was also searched for, but not seen, by ZEUS [10].

A number of recent quark-model estimates exist for $m_{\theta_{c,b}}$. These are based on (i) existing proposals for the structure of the θ , (ii) assumptions about the relation between the structures in the θ and $\theta_{c,b}$ and (iii) the experimental value $m_\theta \simeq 1540$ MeV [11, 12, 13].

In the Jaffe-Wilczek (JW) scenario [11, 18], a structure consisting of two $I = J = 0$, $C = \bar{3}$ qq pairs coupled antisymmetrically to net color 3, with only confinement forces

between the \bar{s} and the qq pairs, is proposed for the θ . The same light quark configuration is then expected for the $\theta_{c,b}$. The analogous baryon splittings, $\simeq m_{\Lambda_{c,b}} - m_{\Lambda}$, are used to estimate $m_{\theta_{c,b}} - m_{\theta}$. The resulting estimates,

$$\begin{aligned} [m_{\theta_c}]_{JW} &\simeq 2710 \text{ MeV} \\ [m_{\theta_b}]_{JW} &\simeq 6050 \text{ MeV} , \end{aligned} \quad (3)$$

lie ~ 100 and 170 MeV, respectively, below $N\bar{D}$ and NB thresholds.

In the Karliner-Lipkin (KL) scenario, the structure proposed for the θ consists of one $I = J = 0$, $C = \bar{3}$ qq pair, as in the JW scenario, but with the spin and color of the remaining pair flipped and anti-aligned to those of the \bar{s} , producing “triquark” ($ud\bar{s}$) quantum numbers $(I, J, C) = (0, \frac{1}{2}, 3)$ [19]. The scenario is motivated by the CM model, in which the hyperfine energy of the KL correlation is lower than that of the JW correlation. The $q\bar{Q}$ hyperfine interactions (which drive the $ud\bar{s}$ cluster formation) are weakened when \bar{s} is replaced by \bar{c}, \bar{b} , reducing the hyperfine attraction in the $\theta_{c,b}$ systems. KL estimate this reduction by assuming that (i) the same diquark-triquark correlation is present in the $\theta_{c,b}$ as in the θ and (ii) the strength of the \bar{Q} hyperfine interactions scale, as in the CM model, with the inverse of the constituent \bar{Q} mass [12]. The resulting modification of the JW estimates leads to

$$\begin{aligned} [m_{\theta_c}]_{JW} &\simeq 2985 \text{ MeV} \\ [m_{\theta_b}]_{JW} &\simeq 6400 \text{ MeV} , \end{aligned} \quad (4)$$

which puts the θ_c and θ_b both $\simeq 180$ MeV above strong decay threshold.

The JW and KL estimates for $m_{\theta_{c,b}}$ rely on the assumption that the change in 1-body energies in going from the θ to $\theta_{c,b}$ is well approximated by the analogous change in the q^3 sector. The systematic uncertainty accompanying this assumption is difficult to estimate. One should bear in mind that, in contrast to the chiral soliton model picture, where a low-lying θ is quite natural [20, 21, 22, 23], the θ mass is much lower than naive constituent quark model expectations would have anticipated. Such expectations are, however, based on effective quark model Hamiltonian which do not explicitly take into account differences in vacuum response in the q^3 and pentaquark sectors.

The dibaryon sector of the bag model provides an illustration of the problems such neglect might produce [24]. The difference between the 1-body energy of a single q^6 bag and that of two isolated q^3 bags, evaluated with standard bag model parameters, is ~ 50 MeV. This relatively small shift, however, results from a close cancellation between two ~ 400 MeV shifts, one in the kinetic, and one in the “zero point” (Z/R) energy. This cancellation is an extremely sensitive function of the bag parameter, $B^{1/4}$ [24]. Sizeable uncertainties are thus present in estimates for the 1-body contribution to splittings between ordinary hadron and multiquark states, even before the relative crudeness of the modelling of the vacuum response in the bag model is taken into account. Typical constituent quark models for which extensions to the $P = +$ pentaquark sector are feasible do not explicitly incorporate even such a simple realization of vacuum response. The resulting estimates for 1-body contributions to pentaquark energies are thus likely

to have sizeable uncertainties. Predictions insensitive to the model treatments of the 1-body energies are thus desirable. We discuss several such predictions in this paper.

The fact that the θ , and hence its partners in the $\overline{10}_F$ multiplet having N and Σ quantum numbers, lie in the midst of the first positive parity excitation baryon region also suggests that past treatments of the excited baryon sector which include only q^3 configurations and neglect mixing with pentaquark states are almost certainly unreliable. The phenomenological successes (or failures) of the models in accounting for the excited baryon spectrum thus need to be revisited. Since the presence of pentaquark configurations makes the phenomenology of the excited baryon sector more complicated than heretofore anticipated, it is useful to have distinctive predictions of the models in the phenomenologically less-complicated exotic sector. The results of this paper, which show significant differences for the *splittings* in the $P = +$ heavy pentaquark sectors of the GB and CM models, provide useful predictions of this type.

In the rest of the paper, we present the results of fully-antisymmetrized GB and CM model calculations for the hyperfine energies of heavy $P = +$, $\bar{Q}\ell^4$ states. The results allow us to investigate, in a dynamical context, cross-cluster interaction and antisymmetrization effects neglected in the JW and KL approaches, and to study the impact of such effects on the JW and KL estimates for the heavy pentaquark hyperfine energies. These points are discussed in Sec. II A. In Sec. II B we discuss the splittings predicted by each model. These are determined up to an overall scale, associated with the size of the $[31]_L$ spatial wavefunction, by the spin-isospin-color structure of the effective interactions, and are independent of the model 1-body energies. This would not be true of the splittings between $\bar{Q}\ell^4$ and $\bar{Q}s^n\ell^{4-n}$ states, where flavor-breaking effects for the problematic 1-body energies would need to be taken into account. In Sec. II B we also present results for the overlaps of the various pentaquark states to NP_H and NV_H^* (with V_H^* the relevant heavy vector meson). Ratios of these overlaps are expected to determine the ratios of effective couplings to the NP_H and NV_H^* decay channels, if the decay mechanism is dominated by “fall-apart” through the p-wave centrifugal barrier [25].

II. HEAVY PENTAQUARK STATES IN THE GB AND CM MODELS

The results which follow are obtained by constructing, in each (I, J_q) channel, all possible color singlet pentaquark states obtainable from fully-antisymmetrized ℓ^4 states with $[31]_L$ orbital and $[211]_c$ color symmetry, and diagonalizing H_{GB} or H_{CM} in the resulting basis. The construction of states and evaluation of matrix elements are standard, and not presented here. Useful cross-checks on the state construction, phase conventions, and spin, color and isospin matrix elements employed are provided by the schematic versions of $H_{GB,CM}$, which neglect the spatial dependence of the operators, allowing the expectations to be obtained by standard group theoretic methods.

In the spatial sector, it is convenient to work with a $[31]_L$ S_4 basis whose members transform as the SS , SA and AS irreps of $S_2^{12} \times S_2^{34}$. The GB and CM light quark hyperfine matrix elements are then completely determined by the spatial matrix elements

$$\langle [31]_L, \rho | f_{GB,CM}(\vec{r}_{12}) | [31]_L, \rho \rangle, \quad \rho = SS, SA, AS \quad (5)$$

To estimate the contributions from the $\bar{Q}\ell$ interactions, which vanish identically only in the $m_{\bar{Q}} \rightarrow \infty$ limit, we employ the following form for the $\rho = SS, SA, AS$ $[31]_L$ spatial wavefunctions (where only the $(L, L_z) = (1, 0)$ component is displayed):

$$\psi_{10}^\rho(\vec{r}_{SS}, \vec{r}_{SA}, \vec{r}_{AS}, \vec{R}_5) = N z_\rho \exp \left[-\frac{\alpha^2}{2} (r_{SS}^2 + r_{SA}^2 + r_{AS}^2) - \frac{\beta^2}{2} R_5^2 \right]. \quad (6)$$

N is a normalization constant, and the relative coordinates, \vec{r}_{SS} , \vec{r}_{SA} , \vec{r}_{AS} , and \vec{R}_5 , are defined in terms of the quark (\vec{r}_i , $i = 1, \dots, 4$) and antiquark (\vec{r}_5) coordinates, by $\vec{R}_5 = \sqrt{\frac{4}{5}} [\frac{1}{4} (\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4) - \vec{r}_5]$, $\vec{r}_{SS} = \frac{1}{2} (\vec{r}_1 + \vec{r}_2 - \vec{r}_3 - \vec{r}_4)/2$, $\vec{r}_{SA} = \frac{1}{\sqrt{2}} (\vec{r}_3 - \vec{r}_4)$, and $\vec{r}_{AS} = \frac{1}{\sqrt{2}} (\vec{r}_1 - \vec{r}_2)$. We quote results below in dimensionless form by removing a factor of $\langle [31]_L, SS | f_{GB,CM}(\vec{r}_{12}) | [31]_L, SS \rangle$ from all hyperfine expectations.

For the GB model, we work with the version employed in Refs. [4, 26], in which GB exchange is considered only between quarks, and not between the light quarks and relevant antiquark, \bar{Q} . The dominant model uncertainty in the predictions for the splittings and overlaps is associated with the α dependence of the ratio, μ_{AS}^{GB} , where

$$\mu_{AS}^{GB,CM} \equiv \langle [31]_L, AS | f_{GB,CM}(\vec{r}_{12}) | [31]_L, AS \rangle / \langle [31]_L, SS | f_{GB,CM}(\vec{r}_{12}) | [31]_L, SS \rangle. \quad (7)$$

For the variationally optimized value of α found in Ref. [4], $\mu_{AS}^{GB} = 0.32$. We allow a $\pm 50\%$ variation about this value to study the possible model dependence of the results.

In the CM model, the residual $\bar{Q}\ell$ interactions present when $m_{\bar{Q}} \neq \infty$ lead both to small shifts in the $m_{\bar{Q}} \rightarrow \infty$ hyperfine expectations, and to additional mixing, in particular between configurations with different light quark spin. This can have a non-trivial impact on the overlaps to NP_H and NV_H^* . The size of these effects depends on β/α . We study this dependence by varying β/α between 0.4 and 0.8. The midpoint of this range corresponds to the variational solution of Ref. [4]. The dominant model dependence for the light quark hyperfine expectations is that associated with μ_{AS}^{CM} . Taking a Gaussian form, $f_{CM}(\vec{r}) = \frac{\sigma^3}{\pi^{3/2}} \exp[-\sigma^2 r^2]$, for $f_{CM}(\vec{r})$, μ_{AS}^{CM} depends on α/σ . This ratio is varied between 0 (zero range limit) and 0.5 (corresponding to a range of $\sim 1/3$ fm). μ_{AS}^{CM} , of course, vanishes in the zero range limit.

A. GB and CM Model Perspectives on the JW and KL Predictions for $m_{\theta_{c,b}}$

The JW $I = J = 0$, $C = \bar{3}$ qq correlation is by far the most attractive qq correlation in the GB model. The version of the GB model employed here, having no $\bar{Q}\ell$ hyperfine interactions, also matches the JW scenario for the $\bar{Q}q$ interactions in both the θ and $\theta_{c,b}$ channels, and hence provides a model context in which the impact of cross-cluster antisymmetrization and interaction effects, neglected in the JW approach, can be investigated. Factoring out C_{GB}/m_ℓ^2 and the qq pair spatial matrix element, $\langle H_{GB} \rangle = -8$ for each $I = J = 0$ qq pair. Two such pairs are thus hyperfine attractive relative to the N , whose expectation is -14 (corresponding to ~ -410 MeV). Once one accounts for cross-cluster interactions and antisymmetrization, the model allows admixtures of other,

higher-lying configurations. corrections, in the $(I, J_\ell, J_q) = (0, 0, \frac{1}{2})$ ground state, is to lower the dimensionless hyperfine expectation from -16 to -21.9 ± 1.3 . As in the JW scenario, the light quark configuration is the same for the θ and $\theta_{c,b}$. The ground state turns out to be rather close to a pure $[4]_{FJ}$ configuration, a result which would be exact in the schematic approximation. The ground state expectation, however, differs significantly from the schematic value, -28 , as a consequence of the reduction of μ_{AS}^{GB} from its schematic limit value, 1.

Although the $I = J = 0$, $C = \bar{3}$ correlation is also the most attractive qq correlation in the CM model, more complicated correlations yield $\bar{Q} = \bar{s}$ pentaquark configurations with hyperfine energies below that of the JW ansatz [19]. The KL diquark-triquark correlation is among these. However, even lower-lying configurations exist [26]. Indeed, for the $P = +$ ground state channel, the same $\bar{Q}\ell$ interactions which lower the KL $ud\bar{s}$ hyperfine expectation also mix the KL and JW configurations. In the θ sector, the lowest eigenvalue, computed using the full set of fully-antisymmetrized states, turns out to be reproduced rather accurately (to within $\sim 1\%$) by a restricted two-channel calculation employing the KL, JW basis which incorporates this mixing but ignores completely cross-cluster antisymmetrization and interaction effects [26]. This suggests that, for the CM model, the θ is dominated by the optimized combination of JW and KL configurations.

The implications of the above discussion for the $\theta_{c,b}$ follow from the $1/m_{\bar{Q}}$ dependence $\bar{Q}\ell$ interactions in the CM model, which effect reduces both the KL triquark hyperfine attraction and the strength of the mixing between JW and KL correlations, relative to the θ . With the conventional constituent quark mass values of Ref. [19], the hyperfine attraction is greater for the JW correlation than the KL correlation, for both the θ_c and θ_b . The KL ansatz, in which the same diquark-triquark correlation posited for the θ is assumed to dominate the $\theta_{c,b}$, thus requires additional dynamics beyond that of the CM model. With strictly CM model interactions, in the $m_{\bar{Q}} \rightarrow \infty$ limit, one expects an $(I, J_q) = (0, 1/2)$ ground state in the $P = +$ sector, dominated by the JW correlation. Neglecting cross-cluster interaction and antisymmetrization effects, as well as mixing with other configurations, the hyperfine expectation for the JW correlation is, after factoring out C_{CM}/m_ℓ^2 and the qq pair spatial matrix element, $\langle H_{CM} \rangle_{JW} = -4$. The true ground state expectation in the model, $\langle H_{CM} \rangle = -3.48 \pm 0.04$, obtained from the full $m_{\bar{Q}} \rightarrow \infty$ limit calculation, indeed occurs for the $(I, J_\ell, J_q) = (0, 0, 1/2)$ channel and is reasonably approximated by $\langle H_{CM} \rangle_{JW}$. The ground state expectations for the \bar{c} and \bar{b} pentaquark channels are in turn well approximated by the $m_{\bar{Q}} \rightarrow \infty$ limit results.

The difference in the hyperfine energies of the θ and $\theta_{c,b}$ in the CM model can be estimated using the N spatial matrix element (which is fixed by the Δ - N splitting) to approximate the corresponding $\theta_{c,b}$ spatial matrix elements. With this estimate, the reductions in the θ_c , θ_b hyperfine energies, relative to the θ , are found to be 125 ± 30 and 129 ± 29 MeV, respectively. Combining these results with the JW/KL estimates for the 1-body contribution to $m_{\theta_{c,b}} - m_\theta$, one obtains the modified estimates

$$\begin{aligned} m_{\theta_c} &\simeq 2835 \pm 30 \text{ MeV} \\ m_{\theta_b} &\simeq 6180 \pm 30 \text{ MeV} , \end{aligned} \tag{8}$$

which put the θ_c just above and the θ_b just below the corresponding strong decay thresh-

olds. The errors quoted in Eqs. (8) reflect only uncertainties in the estimated hyperfine energies. A sizeable additional uncertainty should presumably also be attributed to the baryon-mass-difference-based estimate for the 1-body energy shift between the θ and $\theta_{c,b}$. These uncertainties make a reliable conclusion about the strong interaction stability (or instability) of the $\theta_{c,b}$ in the CM model impossible.

B. Splittings and Decay Overlaps in the Heavy Pentaquark Sector

While uncertainties in the estimates of 1-body energy shifts mean that model predictions for the masses of $P = +$ heavy pentaquark states are subject to considerable uncertainties, the same is *not* true of predictions for the splittings between low-lying excitations and the corresponding $(I, J_q) = (0, 1/2)$ ground state. Up to an overall scale, these splittings are determined by the spin-flavor (or spin-color) structure of H_{GB} (or H_{CM}), and are on the same footing as predictions for the splittings in the ordinary baryon spectrum.

Predictions for $P = +$ heavy pentaquark splittings in the GB and CM models are given in Tables I and II. Column 1 gives the (I, J_q) quantum numbers of the states, Column 2 $\Delta\hat{E}$, the hyperfine splitting relative to the $(I, J_q) = (0, 1/2)$ ground state, in units of $X = C_{GB,CM} \langle [31]_L, SS | f_{GB,CM}(\vec{r}_{12}) | [31]_L, SS \rangle / m_\ell^2$. X is determined by the Δ - N splitting in the limit that the pentaquark SS $ij = 12$ spatial pair expectation is the same as that in the N . Results for the splittings in this limit, ΔE^{est} , are given in Column 3. States with ΔE^{est} greater than $\sim m_\Delta - m_N$ have been omitted from the tables. Deviations of the pentaquark relative-s-wave pair distribution from that in the N will produce only a global rescaling of all splitting values. The remaining columns give the squares of the relative overlaps to NP_H and NV_H^* . These entries are discussed in more detail below. The range given for each entry reflects the impact of varying β/α , α/σ , and μ_{AS}^{GB} within the bounds specified above.

As stressed by Close and Zhao [25], if the dominant mechanism for $P = +$ pentaquark decay to NM ($M = P_H, V_H^*$) is “fall-apart” through the p-wave barrier, the relative widths for the decays $P_1 \rightarrow NM_1$ and $P_2 \rightarrow NM_2$, should be given by

$$\frac{\Gamma[P_1 \rightarrow NM_1]}{\Gamma[P_2 \rightarrow NM_2]} = \frac{\rho_1(m_{P_1})}{\rho_2(m_{P_2})} \left[\frac{\langle NM_1 | P_1 \rangle}{\langle NM_2 | P_2 \rangle} \right]^2, \quad (9)$$

with $\rho_k(m_{P_k})$ the phase space factor for $P_k \rightarrow NM_k$. The overlaps, $\langle NM_k | P_k \rangle$, depend on the structures of the hyperfine eigenstates and the basic spatial overlaps,

$$x_k = \langle N_{123} M_{45} | [31]_L, k \rangle, \quad (10)$$

with $k = SS, SA, AS$. x_{AS} is identically zero as a result of the symmetry of the quark model N spatial wavefunction. In addition, for the phase conventions employed here, $|[31]_L, SS\rangle$ transforms as $P_{23}|[31]_L, SS\rangle = \frac{1}{\sqrt{2}}|[31]_L, SA\rangle - \frac{1}{\sqrt{2}}|[31]_L, AS\rangle$ under the action of the adjacent permutation P_{23} , leading to $x_{SS} = \frac{1}{\sqrt{2}}x_{SA}$. All overlaps can thus be written as a numerical coefficient times the single common spatial overlap factor x_{SA} .

TABLE I: Low-lying positive parity excitations of the $\theta_{c,b}$ in the GB model. All notation is as described in the text.

(I, J_q)	$\Delta\hat{E}$	ΔE^{est} (MeV)	g_P^2	$g_{V^*}^2$
(0,1/2)	0	0	1	3.00
(1,1/2)	4.50→5.71	132→167	2.24→2.54	0.75→0.85
(1,3/2)	4.50→5.71	132→167	0	1.27→1.36
(0,1/2)	10.2→14.5	299→423	2.01→2.07	0.67→0.69
(0,3/2)	10.2→14.5	299→423	0	2.68→2.75

The latter cancels in forming ratios. Columns 4 and 5 of the tables contain, for each excited pentaquark state P^* , the squares of the ratios g_P and g_{V^*} , defined by

$$\begin{aligned} g_P &= \langle NP_H | P^* \rangle / \langle NP_H | P_{gnd} \rangle \\ g_{V^*} &= \langle NV_H^* | P^* \rangle / \langle NP_H | P_{gnd} \rangle , \end{aligned} \quad (11)$$

with P_{gnd} the corresponding $(I, J_q) = (0, 1/2)$ ground state.

Certain general features of the results are evident from the tables. First, for both models, two groups of excited states exist, one with “low” (less than ~ 160 MeV) and one with “high” (comparable to, or slightly less than $m_\Delta - m_N$) excitation energies. Second, the lowest of the excitation energies is significantly smaller in the CM than in the GB model, $\Delta E^{est} \simeq 85 - 90$ MeV versus $\simeq 130 - 160$ MeV. Third, the spectrum of excitations is far denser for the CM model, which has 10 excited states within $\sim m_\Delta - m_N$ of the ground state, compared to only 4 for the GB model. (Even more striking, for the CM model, the 5 “low” excitations and 5 “high” excitations each lie in intervals of size $\sim 50 - 60$ MeV; even if some rescaling of the splitting estimates is required, two regions with a very dense spectrum of states are thus predicted.) Fourth, for the GB model both of the “low” excitations have $I = 1$, whereas low-lying excitations with both $I = 1$ and $I = 0$ exist for the CM model. Fifth, in both models, the “low” excitations have one or both of their overlaps to NP_H or NV_H^* comparable to, or larger than, the ground state overlap to NP_H . As such, if one of the states is experimentally detectable, the others should be as well. Finally, while the “high” excitations in the GB model also have overlaps comparable to that of the ground state, those in the CM model have strongly suppressed overlaps to both NP_H and NV_H^* , with the exception of the lowest of these states, for which the NV_H^* overlap is comparable to the ground state NP_H overlap for some range of the input parameter values. The remainder of the “high” excitations in the CM model should thus decay preferentially to multiparticle final states, making them more challenging to identify experimentally. Note that the range of overlap values is greater for the charm than bottom system in the CM model, reflecting the sensitivity of the overlaps to mixing effects, which are greater for lighter $m_{\bar{Q}}$. The excitation energies are typically much less sensitive, especially so for the “low” group of states.

TABLE II: Low-lying positive parity excitations of the $\theta_{c,b}$ in the CM model. All notation is as described in the text.

Sector	(I, J_q)	$\Delta\hat{E}$	ΔE^{est} (MeV)	g_P^2	g_{V*}^2
Charm	(0,1/2)	0	0	1	0.74→2.22
	(0,1/2)	1.14→1.20	84→88	0.55→1.87	1.54→2.32
	(1,1/2)	1.22→1.47	89→108	1.95→3.41	0.03→0.35
	(0,3/2)	1.29→1.56	94→114	0	1.60→2.79
	(1,3/2)	1.61→1.87	118→137	0	0.85→1.52
	(1,1/2)	1.79→2.07	131→152	0.00→0.14	1.72→2.72
	(0,1/2)	3.08→3.20	226→234	0.29→0.32	0.39→0.92
	(1,1/2)	3.59→3.78	263→276	0.07→0.10	0.00→0.03
	(1,3/2)	3.82→4.10	280→300	0	0.09→0.13
	(0,3/2)	3.84→3.93	281→289	0	0.06→0.21
	(0,1/2)	3.96→4.08	290→298	0.05→0.15	0.09→0.14
Bottom	(0,1/2)	0	0	1.00	1.87→2.71
	(0,1/2)	1.16→1.25	85→92	1.54→2.32	0.88→0.94
	(0,3/2)	1.26→1.35	92→99	0	2.51→3.21
	(1,1/2)	1.43→1.55	105→114	1.76→3.65	0.20→0.76
	(1,3/2)	1.58→1.66	116→122	0	1.36→1.76
	(1,1/2)	1.77→1.99	130→146	0.05→0.46	2.53→2.76
	(0,1/2)	3.06→3.12	224→229	0.32→0.39	0.79→1.10
	(1,1/2)	3.66→3.88	268→284	0.08→0.13	0.02→0.04
	(1,3/2)	3.79→3.98	278→292	0	0.12→0.15
	(0,3/2)	3.91→3.94	286→289	0	0.14→0.25
	(0,1/2)	3.93→3.99	288→292	0.11→0.19	0.07→0.11

III. CONCLUSIONS

We have seen that rather low-lying spin-isospin excitations are expected in the heavy $P = +$ pentaquark sector, in both the GB and CM models. The number of such excitations is especially large for the CM model. A spectrum of excitations richer than in the ordinary baryon sector is generic to the quark model approach to pentaquark states since the number of Pauli-allowed states grows rapidly with the number of constituents. In the $[21]_L$, ℓ^3 ordinary baryon sector, for example, only three channels, with a single allowed state each, are present, to be contrasted to the situation in the pentaquark sector, where 4, 3, 1, 6, 5, 1, 2, 2, and 1 independent states exist for the $[31]_L$, $\bar{Q}\ell^4$ $(I, J_q) = (0, 1/2)$ $(0, 3/2)$, $(0, 5/2)$, $(1, 1/2)$, $(1, 3/2)$, $(1, 5/2)$, $(2, 1/2)$, $(2, 3/2)$ and $(2, 5/2)$ channels, respectively. Spin-orbit partners, which are expected to lie rather nearby in the CM model [27], will make the model spectra even denser.

The pattern of these low-lying excitations has, in addition, been shown to be very different for the two models. Not only is the number of states within ~ 300 MeV of the $(I, J_q) = (0, 1/2)$ ground state much larger for the CM model, but also the minimum excitation energy and pattern of quantum numbers of the “low” group of excitations is significantly different from that of the GB model. Since these states all have an overlap to either NP_H or NV_H^* comparable to or larger than that of the ground state to NP_H , the presence of only $I = 0$ states in the “low” excitation region for the GB model, as well as the presence of such excitation with both $I = 0$ and $I = 1$ in the CM model, should be experimentally detectable by studying two-body decay modes. Experimental results should thus allow one to rule out at least one, and perhaps both, of the models.

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- [1] C. Gignoux, B. Silvestre-Brac and J.M. Richard, Phys. Lett. **B193** (1987) 323; H.J. Lipkin, Phys. Lett. **B195** (1987) 484; J. Leandri and B. Silvestre-Brac, Phys. Rev. **D40** (1989) 2340
 - [2] S. Fleck, C. Gignoux, J.M. Richard and B. Silvestre-Brac, Phys. Lett. **B220** (1989) 616; S. Zouzou and J.M. Richard, Few-Body Syst. **16** (1994) 1
 - [3] L. Ya. Glozman and D. O. Riska, Phys. Rep. **268** (1996) 263
 - [4] F. Stancu, Phys. Rev. **D58** (1998) 111501
 - [5] M. Genovese *et al.*, Phys. Lett. **B425** (1998) 171
 - [6] E.M. Aitala *et al.*, (The E791 Collaboration), Phys. Lett. **B448** (1999) 303
 - [7] T. Nakano, *et al.*, (The LEPS Collaboration), Phys. Rev. Lett. **91** (2003) 012002; V. Barmin, *et al.*, (The DIANA Collaboration), Phys. At. Nucl. **66** (2003) 1715; S. Stepanyan, *et al.*, (The CLAS Collaboration), Phys. Rev. Lett. **91** (2003) 252001; J. Barth, *et al.*, (The SAPHIR Collaboration), Phys. Lett. **B572** (2003) 127; A.E. Asratyan, A.G. Dolgolenko and M.A. Kubantsev, Phys. Atom. Nucl. **67** (2004) 682; V. Kubarovsky *et al.*, (The CLAS Collaboration), Phys. Rev. Lett. **92** (2004) 032001 (Erratum *ibid.* **92** (2004) 049902); H.G. Juengst *et al.*, (The CLAS Collaboration), nucl-ex/0312019; A. Airapetian *et al.*, (The HERMES Collaboration), Phys. Lett. **B585** (2004) 213; A. Aleev *et al.*, (The SVD-2 Collaboration), hep-ex/0401024; M. Abdel-Bary *et al.*, (The COSY-TOF Collaboration), hep-ex/0403011; P.Z. Aslanyan, V.N. Emelyanenko and G.G. Rikhkvitzkaya, hep-ex/0403044; S. Chekanov *et al.*, (The ZEUS Collaboration), Phys. Lett. **B591** (2004) 7; T. Nakano, presentation at NSTAR 2004, Grenoble, France, Mar. 24-27, 2004 (<http://lpsc.in2p3.fr/congres/nstar2004/talks/nakano.pdf>)
 - [8] C. Alt *et al.*, (The NA49 Collaboration), Phys. Rev. Lett. **92** (2004) 042003; K.T. Knopfle *et al.*, (The HERA-B Collaboration), hep-ex/0403020; H.G. Fischer and S. Wenig,

hep-ex/0401014

- [9] A. Aktas *et al.*, (The H1 Collaboration), hep-ex/0403017
- [10] K. Lipka *et al.*, (for the H1 and ZEUS Collaborations), hep-ex/0405051
- [11] R. Jaffe and F. Wilczek, Phys. Rev. Lett. **91** (2003) 232003
- [12] M. Karliner and H.J. Lipkin, hep-ph/0307343
- [13] K. Cheung, Phys. Rev. **D69** (2004) 094029; P.Z. Huang *et al.*, hep-ph/0401191
- [14] J. Rosner, Phys. Rev. **D69** (2004) 094014; X.G. He and X.Q. Li, hep-ph/0403191; T.W. Chiu and T.H. Hsieh, hep-ph/0404007; H.C. Kim, S.H. Lee and Y.S. Oh, hep-ph/0404170
- [15] I.W. Stewart, M.E. Wessling and M.B. Wise, Phys. Lett. **B590** (2004) 185
- [16] B. Wu and B.Q. Ma, hep-ph/0402244; M.A. Nowak, M. Praszalowicz, M. Sadzikowski and J. Wasliuk, hep-ph/0403184
- [17] H.Y. Cheng and C.K. Chua, hep-ph/0403232; Y.R. Liu *et al.*, hep-ph/0404123; K. Cheung, hep-ph/0405281; H.Y. Cheng and C.K. Chua, hep-ph/0406036
- [18] S. Nussinov, hep-ph/0307357
- [19] M. Karliner and H.J. Lipkin, Phys. Lett. **B575**, 249 (2003) and hep-ph/0307243
- [20] A.V. Manohar, Nucl. Phys. **B248** (1984) 19; M. Chemtob, Nucl. Phys. **B256** (1985) 600; M. Praszalowicz in *Skyrmions and Anomalies*, M. Jezabek and M. Praszalowicz eds., World Scientific, 1987 (see also M. Praszalowicz, Phys. Lett.**B575** (2003) 234)
- [21] D. Diakonov, V. Petrov, and M. Polyakov, Z. Phys. **A359** (1997) 305
- [22] H. Weigel, Eur. Phys. J. **A2** (1998) 391; H. Weigel, Proc. Intersections of Particle and Nuclear Physics, hep-ph/0006191; H. Walliser and V.B. Kopeliovich, J. Exp. Theor. Phys. **97** (2003) 433; V.B. Kopeliovich, hep-ph/0310071; D. Borisjuk, M. Faber, A. Kobushkin, hep-ph/0307370 and hep-ph/0312213; D. Diakonov and V. Petrov, Phys. Rev. **D69** (2004) 094011; B. Wu and B.Q. Ma, Phys. Rev. **D69** (2004) 077501 and Phys. Lett. **B586** (2004) 62
- [23] J. Ellis, M. Karliner and M. Praszalowicz, JHEP **0405** (2004) 002
- [24] K. Maltman, Phys. Lett. **B291** (1992) 371
- [25] F.E. Close and Q. Zhao, Phys. Lett. **B590** (2004) 176
- [26] B. Jennings and K. Maltman, Phys. Rev. **D69** (2004) 094020
- [27] J.J. Dudek and F.E. Close, Phys. Lett. **B583** (2004) 278